Quantum diffusion of prices and profits

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Edward W. Piotrowski
Institute of Theoretical Physics, University of Białystok,
Lipowa 41, Pl 15424 Białystok, Poland
e-mail: ep@alpha.uwb.edu.pl
Jan Sładkowski
Institute of Physics, University of Silesia,
Uniwersytecka 4, Pl 40007 Katowice, Poland
e-mail: sladk@us.edu.pl

Abstract

We discuss the time evolution of quotation of stocks and commodities and show that quantum-like correction to the orthodox Bachelier model may be important. Our analysis shows that traders act as a sort of (quantum) tomograph and their strategies can be reproduced from the corresponding Wigner functions. The proposed interpretation of the chaotic movement of market prices imply that Orstein-Uhlenbeck corrections to the Bachelier model should qualitatively matter for large γ scales. We also propose a solution to the currency preference paradox.

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1 Introduction

We have managed to formulate a new approach to quantum game theory [1]-[3] that is suitable for description of market transactions in term of supply and demand curves [4]-[6]. In this approach quantum strategies are vectors in some Hilbert space and can be interpreted as superpositions of trading decisions. Strategies and not the apparatus or installation for actual playing are at the very core of the approach. Spontaneous or institutionalized market transactions are described in terms of projective operation acting on Hilbert spaces of strategies of the traders. Quantum entanglement is necessary to strike the balance of trade. This approach predicts the property of undividity of attention of traders (no cloning theorem) and unifies the English auction with the Vickrey's one attenuating the motivation properties of the latter. There are apparent analogies with quantum thermodynamics that allow to interpret market equilibrium as a state with vanishing financial risk flow. Euphoria, panic or herd instinct often cause violent changes of market prices. Such phenomena can be described by non-commutative quantum mechanics. A simple tactics that maximize the trader's profit on an effective market follows from the model: *accept profits equal or greater then the one you have formerly achieved on average* [5].

The player strategy $|\psi\rangle$ belongs to some Hilbert space and have two important representations $\langle q|\psi\rangle$ (demand representation) and $\langle p|\psi\rangle$ (supply representation) where q and p are logarithms of prices at which the player is buying or selling, respectively [4, 6]. We have defined canonically conjugate hermitian operators (observables) of demand $\mathcal{Q}_k$ and supply $\mathcal{P}_k$ corresponding to the variables q and p characterizing strategy of the k-th player. This led us to the definition of the observable that we call *the risk inclination operator*:

$$H(\mathcal{P}_k, \mathcal{Q}_k) := \frac{(\mathcal{P}_k - p_{k0})^2}{2\,m} + \frac{m\,\omega^2(\mathcal{Q}_k - q_{k0})^2}{2}\,, \qquad (2)$$

where $p_{k0} := \frac{{}_k\langle\psi|\mathcal{P}_k|\psi\rangle_k}{{}_k\langle\psi|\psi\rangle_k}$, $q_{k0} := \frac{{}_k\langle\psi|\mathcal{Q}_k|\psi\rangle_k}{{}_k\langle\psi|\psi\rangle_k}$, $\omega := \frac{2\pi}{\theta}$. θ denotes the characteristic time of transaction [5, 6] which is, roughly speaking, an average time spread between two opposite moves of a player (e. g. buying and selling the same commodity). The parameter $m > 0$ measures the risk asymmetry between buying and selling positions. Analogies with quantum harmonic oscillator allow for the following characterization of quantum market games. One can introduce the constant h_E that describes the minimal inclination of the player to risk. It is equal to the product of the lowest eigenvalue of $H(\mathcal{P}_k, \mathcal{Q}_k)$ and 2θ. 2θ is in fact the minimal interval during which it makes sense to measure the profit.

2 Quantum tomography

Let us consider a simple market with a single commodity $\mathfrak{G}$. A consumer (trader) who buys this commodity measures his/her profit in terms of the variable $\mathfrak{w}=-\mathfrak{q}$. The producer who provides the consumer with the commodity uses $\mathfrak{w}=-\mathfrak{p}$ to this end. Analogously, an auctioneer uses the variable $\mathfrak{w}=\mathfrak{q}$ (we neglect the additive or multiplicative constant brokerage) and a middleman who reduces the store and sells twice as much as he buys would use the variable $\mathfrak{w}=-2\,\mathfrak{p}-\mathfrak{q}$. Various subjects active on the market may manifest different levels of activity. Therefore it is useful to define a standard for the "canonical" variables $\mathfrak{p}$ and $\mathfrak{q}$ so that the risk variable [6] takes the simple form $\frac{\mathfrak{p}^2}{2}+\frac{\mathfrak{q}^2}{2}$ and the variable $\mathfrak{w}$ measuring the profit of a concrete market subject dealing in the commodity $\mathfrak{G}$ is given by

$$u\,\mathfrak{q} + v\,\mathfrak{p} + \mathfrak{w}(u,v) = 0\,, \tag{1}$$

where the parameters u and v describe the activity. The dealer can modify his/her strategy $|\psi\rangle$ to maximize the profit but this should be done within the specification characterized by u and v. For example, let us consider a fundholder who restricts himself to purchasing realties. From his point of view, there is no need nor opportunity of modifying the supply representation of his strategy because this would not increase the financial gain from the purchases. One can easily show by recalling the explicit form of the probability amplitude $|\psi\rangle\in\mathcal{L}^2$ that the triple $(u,v,|\psi\rangle)$ describes properties of the profit random variable $\mathfrak{w}$ gained from trade in the commodity $\mathfrak{G}$. We will use the Wigner function $W(p,q)$ defined on the phase space (p,q)

$$\begin{aligned} W(p,q) \quad &:= \quad h_E^{-1}\int_{-\infty}^{\infty} e^{i\hbar_E^{-1}px}\,\frac{\langle q+\frac{x}{2}|\psi\rangle\langle\psi|q-\frac{x}{2}\rangle}{\langle\psi|\psi\rangle}\,dx \\ &= \quad h_E^{-2}\int_{-\infty}^{\infty} e^{i\hbar_E^{-1}qx}\,\frac{\langle p+\frac{x}{2}|\psi\rangle\langle\psi|p-\frac{x}{2}\rangle}{\langle\psi|\psi\rangle}\,dx, \end{aligned}$$

to measure the (pseudo-)probabilities of the players behaviour implied by his/her strategy $|\psi\rangle$ (the positive constant $h_E=2\pi\hbar_E$ is the dimensionless economical counterpart of the Planck constant discussed in the previous section [4, 6]). Therefore if we fix values of the parameters u and v then the probability distribution of the random variable $\mathfrak{w}$ is given by a partialmarginal distribution $W_{u,v}(w)dw$ that is equal to the Wigner function $W(p,q)$ integrated over the line $u\,p + v\,q + w = 0$:

$$W_{u,v}(w) := \iint\limits_{\mathbb{R}^2} W(p,q)\,\delta(u\,q{+}v\,p+w,0)\,dpdq\,, \tag{2}$$

where the Dirac delta function is used to force the constraint ($\delta(u\,q{+}v\,p+w,0)$). The above integral transform $(W:\mathbb{R}^2 \to \mathbb{R}) \longrightarrow (W:\mathbb{P}^2 \to \mathbb{R})$ is known as the Radon transform [7]. Let us note that the function $W_{u,v}(w)$ is homogeneous of the order -1, that is

$$W_{\lambda u,\lambda v}(\lambda w) = |\lambda|^{-1} W_{u,v}(w)\,.$$

Some special examples of the (pseudo-) measure $W_{u,v}(w)dw$ where previously discussed in [4, 6, 8]. The squared absolute value of a pure strategy in the supply representation is equal to $W_{0,1}(p)$ ($|\langle p|\psi\rangle|^2 = W_{0,1}(p)$) and in the demand representation the relation reads $|\langle q|\psi\rangle|^2 = W_{1,0}(q)$. It is positive definite in these cases for all values of u and v. If we express the variables u and v in the units $\hbar_E^{-\frac{1}{2}}$ the the definitions of $W(p,q)$ and $W_{u,v}$ lead to the following relation between $W_{u,v}(w)$ i $\langle p|\psi\rangle$ or $\langle q|\psi\rangle$ for both representations[1][9]:

$$W_{u,v}(w) = \frac{1}{2\pi|v|}\Bigl|\int_{-\infty}^{\infty} \mathrm{e}^{\frac{\mathrm{i}}{2v}(up^2+2pw)} \langle p|\psi\rangle\,dp\,\Bigr|^2. \tag{3}$$

The integral representation of the Dirac delta function

$$\delta(uq{+}vp{+}w,0) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \mathrm{e}^{\mathrm{i}k(uq+vp+w)}dk \tag{4}$$

helps with finding the reverse transformation to (2). The results is:

$$W(p,q) = \frac{1}{4\pi^2}\iiint\limits_{\mathbb{R}^3} \cos(uq{+}vp{+}w)\,W_{u,v}(w)\,dudvdw\,. \tag{5}$$

[1]One must remember that switching roles of p and q must be accompanied by switching u with v

Traders using the same strategy (or single traders that can adapt his moves to variable market situations) can form tomographic pictures of their strategies by measuring profits from trading in the commodity $\mathfrak{G}$. These pictures would be influenced by various circumstances and characterized by values of u and v. These data can be used for reconstruction of the respective strategies expressed in terms the Wigner functions $W(p,q)$ according to the formula (5).

2.1 Example: marginal distribution of an adiabatic strategy

Let us consider the Wigner function of the n-th excited state of the harmonic oscillator [10]

$$W_n(p,q)dpdq = \frac{(-1)^n}{\pi\hbar_E}\, e^{-\frac{2H(p,q)}{\hbar_E\omega}}\, L_n\big(\frac{4H(p,q)}{\hbar_E\omega}\big)dpdq\,,$$

where L_n is the n-th Laguerre polynomial. We can calculate (cf. the definition (2)) marginal distribution corresponding to an adiabatic strategy. The identity [11]

$$\int_{-\infty}^{\infty} \mathrm{e}^{\mathrm{i}kw-\frac{k^2}{4}}\, L_n\Big(\frac{k^2}{2}\Big)\, dk = \frac{2^{n+1}\sqrt{\pi}}{n!}\, \mathrm{e}^{-w^2} H_n^2(w)\,,$$

where $H_n(w)$ are the Hermite polynomials, Eq. (4) and the generating function for the Laguerre polynomials, $\frac{1}{1-t}\,\mathrm{e}^{\frac{xt}{t-1}} = \sum_{n=0}^{\infty} L_n(x)\, t^n$ lead to

$$W_{n,u,v}(w) = \frac{2^n}{\sqrt{\pi(u^2+v^2)}\, n!}\; \mathrm{e}^{-\frac{w^2}{u^2+v^2}}\, H_n^2\Big(\frac{w}{\sqrt{u^2+w^2}}\Big) = |\langle w|\psi_n\rangle|^2. \quad (6)$$

This is the squared absolute value of the probability amplitude expressed in terms of the variable w. It should be possible to interpret Eq. (6) in terms of stochastic interest rates but this outside the scope of the present paper.

3 Canonical transformations

Let us call those linear transformations $(\mathcal{P},\mathcal{Q})\to(\mathcal{P}',\mathcal{Q}')$ of operators $\mathcal{P}$ and $\mathcal{Q}$ that do not change their commutators $\mathcal{P}\mathcal{Q}-\mathcal{Q}\mathcal{P}$ canonical. The canonical transformations that preserve additivity of the supply and demand components of the risk inclination operator $\frac{\mathcal{P}^2}{2m}+\frac{m\mathcal{Q}^2}{2}$ [4, 6] can be expressed in the compact form

$$\begin{pmatrix}\mathcal{P}\\ \mathcal{Q}\end{pmatrix}=\begin{pmatrix}\frac{\operatorname{Re}z}{z\overline{z}} & \operatorname{Im}z\\ -\frac{\operatorname{Im}z}{z\overline{z}} & \operatorname{Re}z\end{pmatrix}\begin{pmatrix}\mathcal{P}'\\ \mathcal{Q}'\end{pmatrix}, \tag{7}$$

where $z\in\overline{\mathbb{C}}$ is a complex parameter that is related to the risk asymmetry parameter m, $m=z\overline{z}$. Changes in the absolute value of the parameter z correspond to different proportions of distribution of the risk between buying and selling transactions. Changes in the phase of the parameter z may result in mixing of supply and demand aspects of transactions. For example, the phase shift $\frac{\pi}{4}$ leads to the new canonical variables $\mathcal{P}'=\mathcal{Y}:=\frac{1}{\sqrt{2}}(\mathcal{P}-\mathcal{Q})$ and $\mathcal{Q}'=\mathcal{Z}:=\frac{1}{\sqrt{2}}(\mathcal{P}+\mathcal{Q})$. The new variable $\mathcal{Y}$ describes arithmetic mean deviation of the logarithm of price from its expectation value in trading in the asset $\mathfrak{G}$. Accordingly, the new variable $\mathcal{Z}$ describes the profit made in one buying-selling cycle in trading in the asset $\mathfrak{G}$. Note that the normalization if forced by requirement of canonicality of transformations. In the following we will use Schrödinger-like picture for description of strategies. Therefore strategies will be functions of the variable y being the properly normalized value of the logarithm of the market price of the asset in question. The dual description in terms of the profit variable z is also possible and does not require any modification due to the symmetrical form of the risk inclination operator $H(\mathcal{Y},\mathcal{Z})$ [4, 6]. The player's strategy represents his/her actual position on the market. To insist on a distinction, we will define tactics as the way the player decides to change his/her strategy according to the acquired information, experience and so on. Therefore, in our approach, strategies are represented by vectors in Hilbert space and tactics are linear transformations acting on strategies (not necessary unitary because some information can drastically change the players behaviour!)

4 Diffusion of prices

Let us consider an analogue of canonical Gibbs distribution function

$$\mathrm{e}^{-\gamma H(\mathcal{Y},\mathcal{Z})}, \tag{8}$$

where we have denoted the Lagrange multiplier by γ instead of the more customary β for later convenience. The analysis performed in Ref. [12] allows to interpret (8) as non-unitary tactics (say thermal tactics) leading to a new strategy according to the results of Ref. [13]. Classical description of the time evolution of a logarithm of price of an asset is known as the Bachelier model. This model is based on supposition that the probability density of the logarithm of price fulfills a diffusion equation with an arbitrage forbidding drift. Therefore we will suppose that the (quantum) expectation value of the arithmetic mean of the logarithm of price of an asset $E(\mathcal{Y})$ is a random variable described by the Bachelier model. So the price variable y has the properties of a particle performing random walk that can be described as Brown particle at large time scales t and as Rayley particle at short time scales γ [14]. The superposition of these two motions gives correct description of the behaviour of the random variable y. It seems that the parameters t and γ should be treated as independent variables because the first one parameterizes evolution of the "market equilibrium state" and the second one parameterizes the "quantum" process of reaching the market equilibrium state [15, 16]. Therefore the parameter γ can be interpreted as the inverse of the temperature of a canonical portfolio (cf. Ref.[12]) that represents strategies of traders having the same risk inclination. These traders adapt the tactics that so that the resulting strategy form a ground state of the risk inclination operator $H(\mathcal{Y},\mathcal{Z})$. Regardless of the possible interpretations, adoption of the tactics (8) means that traders have in view minimization of the risk (within the available information on the market). It is convenient to adopt such a normalization (we are free to fix the Lagrange multiplier) of the operator of thermal tactics so that the resulting strategy is its fixed point. This normalization preserves the additivity property, $\mathcal{R}_{\gamma_1+\gamma_2}=\mathcal{R}_{\gamma_2}\mathcal{R}_{\gamma_1}$ and allows consecutive (iterative) implementing of the tactics. The operator of thermal tactics takes the form ($\omega=\hbar_E=1$)

$$\mathcal{R}_\gamma := \mathrm{e}^{-\gamma(H(\mathcal{Y},\mathcal{Z})-\frac{1}{2})}\,.$$

Note that the operator $H(\mathcal{Y},\mathcal{Z})-\frac{1}{2}$ annihilate the minimal risk strategy. The integral representation of the operator $\mathcal{R}_\gamma$ (heat kernel) acting on strategies $\langle y|\psi\rangle\in\mathcal{L}^2$ reads:

$$\langle y|\mathcal{R}_\gamma\psi\rangle = \int_{-\infty}^{\infty}\mathcal{R}_\gamma(y,y')\langle y'|\psi\rangle dy', \tag{9}$$

where (the Mehler formula [17])

$$\mathcal{R}_\gamma(y,y') = \frac{1}{\sqrt{\pi(1-\mathrm{e}^{-2\gamma})}}\,\mathrm{e}^{-\frac{y^2-y'^2}{2}-\frac{(\mathrm{e}^{-\gamma}y-y')^2}{1-\mathrm{e}^{-2\gamma}}}\,.$$

$\mathcal{R}_\gamma(y,y')$ gives the probability density of Rayleigh particle changing its velocity from y' to y during the time γ. Therefore the fixed point condition for the minimal risk strategy takes the form

$$\int_{-\infty}^{\infty}\mathcal{R}_\gamma(y,y')\,\mathrm{e}^{\frac{y^2-y'^2}{2}}dy' = 1\,.$$

¿From the mathematical point of view, the tactics $\mathcal{R}_\gamma$ is simply an Orstein-Uhlenbeck process. It is possible to construct such a representation of the Hilbert space $\mathcal{L}^2$ so that the fixed point of the thermal tactics corresponds to a constant function. This is convenient because the "functional" properties are "shifted" to the probability measure $\widetilde{dy} := \frac{1}{\sqrt{\pi}}\,\mathrm{e}^{-y^2}dy$. After the transformation $\mathcal{L}^2(dy)\rightarrow\mathcal{L}^2(\widetilde{dy})$, proper vectors of the risk inclination operator are given by Hermite polynomials (the transformation in question reduces to the multiplication of vectors in $\mathcal{L}^2$ by the function $\sqrt[4]{\pi}\,\mathrm{e}^{\frac{y^2}{2}}$). Now Eq.(9) takes the form:

$$\widetilde{\langle y|\mathcal{R}_\gamma\psi\rangle} = \int_{-\infty}^{\infty}\widetilde{\mathcal{R}}_\gamma(y,y')\,\widetilde{\langle y'|\psi\rangle}\,\widetilde{dy'}\,,$$

where

$$\widetilde{\mathcal{R}}_\gamma(y,y') := \frac{1}{\sqrt{1-\mathrm{e}^{-2\gamma}}}\,\mathrm{e}^{y'^2-\frac{(\mathrm{e}^{-\gamma}y-y')^2}{1-\mathrm{e}^{-2\gamma}}}\,.$$

In this way we get the usual description of the Orstein-Uhlenbeck process in terms of a kernel $\widetilde{\mathcal{R}}_\gamma(y,y')$ being a solution to the Fokker-Planck equation [18].

5 "Classical" picture of quantum diffusion

Let us consider the integral kernel of one-dimensional exponent of the Laplace operator $\mathrm{e}^{-\frac{\gamma}{2}\frac{\partial^2}{\partial y^2}}$ representing the fundamental solution of the diffusion equation

$$\frac{\partial f(y,\gamma)}{\partial\gamma} = \frac{1}{2}\,\frac{\partial^2 f(y,\gamma)}{\partial y^2}\,.$$

The kernel takes the following form

$$\mathcal{R}^0_\gamma(y,y') := \frac{1}{\sqrt{2\pi\gamma}}\,\mathrm{e}^{-\frac{(y-y')^2}{2\gamma}}\;,$$

and the appropriate measure invariant with respect to $\mathcal{R}^0_\gamma(y,y')$ reads:

$$dy_0 := \frac{1}{\sqrt{\pi\gamma}}\,\mathrm{e}^{-\frac{y^2}{2\gamma}}\,dy\,.$$

The corresponding stochastic process is known as the Wiener-Bachelier process. In physical applications the variables y and γ are interpreted as position and time, respectively (Brownian motion). Let us define the operators $\mathcal{X}_k$ acting on $\mathcal{L}^2$ as multiplications by functions $x_k(y(\gamma_k))$ for successive steps $k=1,\ldots,n$ such that $-\frac{\gamma}{2}\leq\gamma_1\leq\ldots\leq\gamma_n\leq\frac{\gamma}{2}$. The corresponding (conditional) Wiener measure $dW^\gamma_{y,y'}$ for $y=y(-\frac{\gamma}{2})$ and $y'=y(\frac{\gamma}{2})$ is given by the operator

$$\int\prod_{k=1}^{n}x_k(y(\gamma_k))\,dW^\gamma_{y,y'} := \left(\mathrm{e}^{-\frac{\gamma_1+\gamma/2}{2}\frac{\partial^2}{\partial y^2}}\mathcal{X}_1\mathrm{e}^{-\frac{\gamma_2-\gamma_1}{2}\frac{\partial^2}{\partial y^2}}\mathcal{X}_2\cdots\mathcal{X}_n\mathrm{e}^{-\frac{\gamma/2-\gamma_n}{2}\frac{\partial^2}{\partial y^2}}\right)(y,y')\,.$$

If the operators $\mathcal{X}_k$ are constant ($x_k(y(\gamma_k))\equiv 1$) then

$$\int dW^\gamma_{y,y'} = \mathcal{R}^0_\gamma(y,y')\,.$$

The Wiener measure allows to rewrite the integral kernel of the thermal tactics in the form [17]

$$\mathcal{R}_\gamma(y,y') = \int\mathcal{T}'\,\mathrm{e}^{-\int\limits_{-\gamma/2}^{\gamma/2}\frac{y^2(\gamma')-1}{2}\,d\gamma'}\,dW^\gamma_{y,y'} \tag{10}$$

known as the Feynman-Kac formula where $\mathcal{T}'$ is the anti-time ordering operator. According to the quantum interpretation of path integrals [19] we can expand the exponent function in Eq. 10 to get "quantum" perturbative corrections to the Bachelier model that result interference of all possible classical scenarios of profit changes in time spread γ, cf. [20].[2] These

[2]Note that in the probability theory one measures risk associated with a random variable by squared standard deviation. According to this we could define the complex profit operator $\mathcal{A} := \frac{1}{\sqrt{2}}(\mathcal{Y}+\mathrm{i}\,\mathcal{Z})$. The appropriate risk operator would take the form $H(\mathcal{A}^\dagger,\mathcal{A}) = \mathcal{A}^\dagger\mathcal{A}+\frac{1}{2}$.

quantum corrections are unimportant for short time intervals $\gamma \ll 1$ and the Orstein-Uhlenbeck process resembles the Wiener-Bachelier one. This happens, for example, for "high temperature" thermal tactics and for disorientated markets (traders). In effect, due to the cumulativity of dispersion during averaging for normal distribution $\eta(x, \sigma^2)$

$$\int_{-\infty}^{\infty} \eta(x+y, \sigma_1^2)\, \eta(y, \sigma_2^2)\, dy = \eta(x, \sigma_1^2 + \sigma_2^2)$$

the whole quantum random walk parameterized by γ can be incorporated additively into mobility parameter of the classical Bachelier model. This explain changes in mobility of the logarithm of prices in the Bachelier model that follow, for example, from changes in the tactics temperature or received information. From the quantum point of view the Bachelier behaviour follows from short-time tactics adopted by the rest of the world considered as a single trader [4]. Collected information about the market results after time $\gamma \ll 1$ in the change of tactics that should lead the trader the strategy being a ground state of the risk inclination operator (localized in the vicinity of corrected expectation value of the price of the asset in question). This should be done in such a way that the actual price of the asset is equal to the expected price corrected by the risk-free rate of return (arbitrage free martingale)[21]. Both interpretations of the chaotic movement of market prices imply that Orstein-Uhlenbeck corrections to the Bachelier model should qualitatively matter only for large γ scales.

6 Final remarks

Our analysis shows that traders dealing in the asset $\mathfrak{G}$ act as a sort of (quantum) tomograph and their strategies can be reproduced from the corresponding Wigner functions. It might happen that the experience acquired in medicine, geophysics and radioastronomy would be used to investigate intricacies of supply and demand curves. An attentive reader have certainly noticed that we have supposed that the drift of the logarithm of the price of an asset must be a martingale (that typical of financial mathematics [21]). Now suppose that we live in some imaginary state where the ruler is in a position to decree the exchange rate between the local currency $\mathfrak{G}$ and some other currency $\mathfrak{G}'$. The value of the logarithm of the price of $\mathfrak{G}$ (denoted by $\mathfrak{n}$) is proportional to the result of measurement of position of a one dimensional Brown particle. Any owner of $\mathfrak{G}$ will praise the ruler for such policy and prefer $\mathfrak{G}$ to $\mathfrak{G}'$ because the the price of $\mathfrak{G}$ in units of $\mathfrak{G}'$ will, on average, raise (the process $\exp \mathfrak{n}$ is sub-martingale). For the same reasons a foreigner will be content with preferring $\mathfrak{G}'$ to $\mathfrak{G}$. This currency preference paradoxical property of price drifts suggest that the common assumption about logarithms of assets prices being a martingale should be carefully analyzed prior to investment. If one measures future profits from possessing $\mathfrak{G}$ with the anticipated change in quotation of $\mathfrak{n}$ then the paradox is solved and expectation values of the profits from possessing $\mathfrak{G}$ or $\mathfrak{G}'$ are equal to zero (cf. Bernoulli's solution to the Petersburg paradox [21]). Note that if we suppose that the price of an asset and not its logarithms is a martingale then the proposed model of quantum price diffusion remains valid if we suppose that the observer's reference system drifts with a suitably adjusted constant velocity (in logarithm of price variable).

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