Quantum Auctions: Facts and Myths ⋆

(RePEc:sla:eakjkl:29 10-II-2008)

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Abstract

Quantum game theory, whatever opinions may be held due to its abstract physical formalism, have already found various applications even outside the orthodox physics domain. In this paper we introduce the concept of a quantum auction, its advantages and drawbacks. Then we describe the models that have already been put forward. A general model involves Wigner formalism and infinite dimensional Hilbert spaces – we envisage that the implementation might not be an easy task. But a restricted model advocated by the Hewlett-Packard group (Hogg et al) seems to be much easier to implement. We focus on problems related to combinatorial auctions and technical assumptions that are made. Powerful quantum algorithms for finding solutions would extend the range of possible applications. Quantum strategies, being qubits, can be teleported but are immune from cloning – therefore extreme privacy of agent's activity could in principle be guaranteed. Then we point out some key problem that have to be solved before commercial use would be possible. With present technology, optical networks, single photon sources and detectors seems to be sufficient for experimental realization in the near future.

Key words: quantum game, quantum information theory, quantum markets, quantum finance, auction theory
PACS: 02.50.Le, 03.67.Lx, 05.50.+q, 05.30.d

⋆ This research was supported in part by the **Polish Ministry of Science and Higher Education project No N519 012 31/1957**.

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Motto:

Combinatorial auctions[1] *are the great frontier of auction theory today, ...*
Roger B. Myerson on the back cover of Peter Cramton, Yoav Shoham and Richard Steinberg (Eds.), Combinatorial Auctions, MIT Press, Cambridge, 2006.

1 Introduction

Quantum game theory [1] - [4] emerged as an abstract idea in quantum theory but soon it was realized that it offers powerful analytical tools that might be used outside physical laboratories. Game theory, the study of decision making in conflict situations, seems to have asked for a quantum version. For example, games against Nature include those for which Nature is quantum mechanical. But does quantum theory offer more subtle ways of playing games? Game theory considers strategies that are probabilistic mixtures of pure strategies. Why can not they be intertwined in a more complicated way, for example interfered or entangled? The research already performed suggests that there are several possible niches [2] for the quantum game products launch. The most promising seem to be quantum cryptography, "quantum" hazard and quantum auctions. One can already buy quantum cryptographic equipment provided by *id Quantique, MagiQ Technologies, SmartQuantum* and, what is more important, many well known industrial concerns have revealed their interest in quantum technologies, not to mention military/security oriented projects. Although the market may be worth of billions dollar the involved complication obstruct massive application. This would certainly change if the security of the presently used cryptographic systems is challenged by the increase in computational power or other developments. Quantum hazard has big potential and it seems that present technology is sufficient for implementation. The implementation will be costly, but if you compare the estimated costs of the order of $\$10^8$ with the amount of money spent on advertising related products the situation seems to be promising! Optical cluster states presently form the most promising implementation environment [5,6]. The key issue is to invent a simple to implement, possibly interesting (drawing in), quantum game – the inventor would get the due gratification! The first, to our knowledge, proposal

[1] That is auctions, where bidders can bid on combinations of items. They lead to more efficient allocations than in traditional multi-item auctions where the agents' valuations of the items are not additive. However, determining the winners so as to maximize revenue is usually a NP-complete computational problem.

[2] The point is that niches should not be equated with small but rather you should think of narrow: the targeting at a more narrowly defined customer group seeking a distinctive source of benefits. Niche markets are not the marginal opportunity that they once used to be.

was put forward in ref. [7]. Although it is implementable, one can hardly say it would be exciting for non-physicists. On the other hand, quantum auctions, if ever implemented, would be designed for very specific and limited business circles: the volume must be huge and items combined. The paper is organized as follows. We will begin by presenting the general idea of a quantum game and methods of gaining an advantage over "classical opponent". Then we will attempt give a definition of a quantum auction and review problems that have already been discussed in the literature. Finally we will try to show some problems that should be addressed in the near future. In the following discussion we will use quantum auction theory as a formal theoretical tool but the broadcasted message would be that it would probably be used massive combinatorial auctions in the future [8] or in compound securities trading [9].

2 Quantum games

It is not easy to give the precise date of birth of quantum game theory. Quantum games have been with us camouflaged since the very beginning of the quantum era because a lot of experiments can be reformulated in terms of game theory. Quantum game theory began with works of Wiesner on quantum money [10], Vaidman, who probably first used the term game in quantum context [11] and Meyer [1] and Eisert et al [2] who first formulated their problems in game theory formalism. Possible applications of quantum games in biology are thoroughly discussed by Iqbal [12], in economics by Piotrowski and Sładkowski [13,14]. Flitney and Abbott quantized Parrondo's paradox [15]. The most popular experimental realizations are described in refs [16,6]. In principle, any quantum system that can be manipulated by at least one party and where the utility of the moves can be reasonably defined, quantified and ordered may be conceived as a quantum game [3]. The quantum system may be referred to as a *quantum board* although the term *universum of the game* seems to be more appropriate [17]. Usually one supposes that all players know the state of the game at the beginning and at some crucial stages that depend an the game being played. This is a subtle point because it is not always possible to identify the state of a quantum system (one can easily give examples of systems that are only partially accessible to some players [18]). A "realistic" quantum game should include measuring apparatuses or information channels that provide information on the state of the game at crucial stages and specify the way of its termination. Therefore we will suppose that a *two–player quantum game* $\Gamma = (\mathcal{H}, \rho, S_A, S_B, P_A, P_B)$ is completely specified by the underlying Hilbert space $\mathcal{H}$ of the physical system, the initial state $\rho \in \mathcal{S}(\mathcal{H})$, where $\mathcal{S}(\mathcal{H})$ is the associated state space, the sets S_A and S_B of permissible quantum operations of the two players, and the pay–off

[3] One can also consider the class of games against Nature.

(utility) functions P_A and P_B, which specify the pay–off for each player. A *quantum strategy* $s_A \in S_A$, $s_B \in S_B$ is a collection of admissible quantum operations, that is the mappings of the space of states onto itself. One usually supposes that they are completely positive trace preserving maps. The quantum game's definition may also include certain additional rules, such as the order of the implementation of the respective quantum strategies or restriction on the admissible communication channels, methods of stopping the game etc. The generalization for the N players case is obvious. Schematically we have:

$$\rho \mapsto (s_A, s_B, \ldots) \mapsto \sigma \Rightarrow (P_A, P_B, \ldots)\,,$$

where σ denote the measurement of state of the game combined with the prize allocation algorithm.

3 Quantum auctions

Quantum auction are quantum games designed for goods allocations. There is hope that due to the quantum computation speed up resulting from unitary evolution and entanglement they might be some day an alternative for "classical" auctions in cases where combinatorial and computational problems hinder the designers in their work. Extreme security and privacy are other strong points of quantum auctions. Currently, it is difficult to find out if this is a feasible task. Bellow, we describe some proposals that have already been put forward. Shortly, a protocol for a quantum auction should specify the following steps.

- Auctioneer specifies conventional "classical" details of the auction such as the schedule, goods to be sold etc.
- Auctioneer specifies the implementation of the quantum auction.
- Auctioneer specifies the initial state distribution, implementation of strategies and main features of the search algorithms to be used (eg probabilistic, deterministic etc).
- Search for the winners and good allocations (this process might be repeated several times).
- Methods of goods delivery and clearing.

The first and the last items are not directly connected with the "quantumness" of the auction and will not be discussed here. Schematically we can write

$$\rho \mapsto (s_1, s_2, \cdots, s_n) \mapsto \sigma \Rightarrow (P_1, \cdots, P_n)\,,$$

where s_i and P_i denote bids and goods allocation, respectively.

3.1 GG model

An interesting model of, roughly speaking, a population of N quantum bargaining games being played on a market was recently proposed by Gonçalves and Gonçalves [19]. The idea behind it is that one can introduce a "population numbers" $n_1, n_2, \ldots, n_m$ for all alternative strategies combinations. This fact is described by bosonic creation and annihilation operators $a_k^\dagger$ and a_k with standard commutation relation. The number of all possible combination, $m = \prod_k N_k$ is unlimited (N_j is the number of alternative strategies for the j-th player. In general, the j-th agent strategy profile is $|p_j\rangle = \sum_i c_i |s_i(p_j)\rangle$, where c_i is the probability amplitude of strategy s_i. The unitary evolution of the strategy state $|p_j, t_{fin}\rangle = U(t_{fin}, t_{ini})|p_j, t_{ini}\rangle$ is governed by a unitary operator of the form

$$U(t_{fin}, t_{ini}) = \prod_{k=0}^{k_{fin}} U(t_{k+1}, t_k),$$

where k parameterizes the $k_{fin} + 1$ trading rounds. In a simplified single-asset model, where there are only two strategies (buying and selling) for each agent the state $|n_0, n_1\rangle$ is characterized by two occupation numbers n_0 and n_1 giving the number of agents that are selling and buying, respectively. Then the unitary evolution for the k-th trading round can be given in the following form:

$$U(t_{k+1}, t_k) = \exp(\sum_{j=0}^{1}(\xi_j(k, \tau_k)a_j^\dagger - \xi_j(k, \tau_k)^* a_j)),$$

where τ_k is the duration of each trading round, $\xi_j(k, \tau_k) = -i\tau_k\mu_j(k)$ with $\mu_j(k)$ a game-dependent real number that incorporates the cognitive dynamics. Even this oversimplified model reproduces multifractal signatures similar to those of real markets [19]. This is an interesting analytical tool – the model can be run online at the web [4]. Experimental implementation would not be easy, but the mastering of coherent photon states might render it feasible.

3.2 PS model

Piotrowski and Sładkowski [13] put forward a quantum model of bargaining in an infinite dimensional Hilbert space. The k-th agent strategies $|p_j\rangle_k$ belong to Hilbert spaces H_k. The initial state of the game $|\Psi\rangle_{in} := \sum_k |\psi\rangle_k$ is a vector in the direct sum of Hilbert spaces of all players, $\oplus_k H_k$. Then one defines canonically conjugate hermitian operators of demand $\mathcal{Q}_k$ and supply $\mathcal{P}_k$ for each Hilbert space H_k analogously to their physical counterparts, position and

[4] http://ccl.northwestern.edu/netlogo/models/community/Quantum_Financial_Market

momentum. The observable

$$H(\mathcal{P}_k, \mathcal{Q}_k) := \frac{(\mathcal{P}_k - p_{k0})^2}{2m} + \frac{m\omega^2(\mathcal{Q}_k - q_{k0})^2}{2}, \tag{12}$$

where $p_{k0} := \frac{{}_k\langle\psi|\mathcal{P}_k|\psi\rangle_k}{{}_k\langle\psi|\psi\rangle_k} \neq E(\mathcal{P}_k)$, $q_{k0} := \frac{{}_k\langle\psi|\mathcal{Q}_k|\psi\rangle_k}{{}_k\langle\psi|\psi\rangle_k}$, $\omega := \frac{2\pi}{\theta}$, and is called *the risk inclination operator*, cf Ref. [20]. θ denotes the characteristic time of transaction introduced in the MM model [21]. Noncommuting variables appear in a natural way here [20]. A transaction consists in a transition from the state of traders strategies $|\Psi\rangle_{in}$ to the one describing the capital flow state $|\Psi\rangle_{out} := \mathcal{T}_\sigma|\Psi\rangle_{in}$, where $\mathcal{T}_\sigma := \sum_{k_d} |q\rangle_{k_d\,k_d}\langle q| + \sum_{k_s} |p\rangle_{k_s\,k_s}\langle p|$ is the projective operator defined by the division σ of the set of traders $\{k\}$ into two separate subsets $\{k\} = \{k_d\} \cup \{k_s\}$, the ones buying at the price $e^{q_{k_d}}$ and the ones selling at the price $e^{-p_{k_s}}$ in the round of transactions in question. The key role is played by a (quantum) algorithm $\mathcal{A}$ that determines the division σ of the market, the set of price parameters $\{q_{k_d}, p_{k_s}\}$ and the values of capital flows. The capital flows resulting from an ensemble of simultaneous transactions correspond to the physical process of measurement. The later are settled by the distribution

$$\int_{-\infty}^{\ln c} \frac{|\langle q|\psi\rangle_k|^2}{{}_k\langle\psi|\psi\rangle_k} dq$$

which is interpreted as the probability that the trader $|\psi\rangle_k$ is willing to buy the asset $\mathfrak{G}$ at the transaction price c or lower. In an analogous way the distribution

$$\int_{-\infty}^{\ln \frac{1}{c}} \frac{|\langle p|\psi\rangle_k|^2}{{}_k\langle\psi|\psi\rangle_k} dp$$

gives the probability of selling $\mathfrak{G}$ by the trader $|\psi\rangle_k$ at the price c or greater.

These probabilities are in fact conditional because they describe the situation after the division σ is completed. Various possible class of tactics and strategies are discussed in ref. [13]. Experimental implementation would be hard but quantum markets would have such astonishing features the that it is worth a try.

3.3 The HP group model

In this model any possible price of each item (multiply auctions are possible) are encoded in strings of qubits [25]-[26]. The bidder specifies her bid by selecting the corresponding vector of the Hilbert space – each bidder gets p, $p = p_{item} + p_{price}$ qubits and can only operate on those bits. Thus each bidder has 2^p possible bids values, and can create superpositions of these bids: for multiply item auction the bid is a superposition $\sum_j \alpha_j |bundle_j\rangle \otimes |price_j\rangle$ for each bundle of items. A superposition of bids specifies a set of distinct bids, with at most one allowed to win and amplitudes of the superposition correspond to the likelihood of various outcomes for the auction. The protocol uses a distributed adiabatic search that guarantee that bidder's strategies remain private [27]. The search operation processing input from the bidders implemented by unitary operators, giving the overall operator $U = U_1 \otimes U_2 \otimes \ldots \otimes U_n(1)$, where n is the number of bidders and U_i the operator of i-th bidder [25]. A perfect search is not always possible and probabilistic goods allocations should be admitted. Some additional sub-procedures might be necessary to prevent dishonest agents and auctioneers from getting advantage [26]. In such a "brute force" implementation the existence of equilibria can be proved. This proposal seems to be the easiest to implement and especially suitable for combinatorial auctions. Details and simulations are given in Refs [25]-[28].

4 Conclusion

Quantum auctions are certainly an interesting theoretical alternative for complex and massive auctions but are they feasible? Encoding bids in quantum states is a challenge to (quantum) game theory: quantum auctions would almost always be probabilistic and may provide us with specific incentive mechanisms etc. As the outcome may depend on amplitudes of quantum strategies sophisticated apparatus and specialist may be necessary. Therefore, we envisage some changes in the law and habits. Combinatorial auctions seem to be the most promising field. But despite the promising quantum-like experiments [28] commercial implementation of quantum auctions is a demanding challenge that would hardly be accomplished without a major technological breakthrough in mastering quantum devices. Recent development in quantum information processing raises many important issues [29]: *Are markets predestined to quantum technologies? Would we rise to the challenge?* For the present, quantum game theory is only an interesting theoretical tool in various fields of research but the situation might soon change in a dramatic way.

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