What was the temperature of the Bagsik financial oscillator?

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Abstract

We argue that the recently published by Przystawa and Wolf model of the Bagsik financial oscillator is oversimplified and unrealistic. We propose and analyze a refined explanation of this rare financial phenomenon. We have found an example that results in profitability about 45 000 times bigger than that of the Przystawa and Wolf model.

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1 Introduction

In a recent paper [1] Przystawa and Wolf discuss an algorithm (denoted below as sbO) that, if exploited in the plunged in hyper-inflation Poland of the early nineties, should bring enormous profits. This algorithm make the most of constancy of the exchange ratio between two currencies, dollar (USD) and Polish złoty (PLN), and interest rates. Przystawa and Wolf suggest that Polish contractor Bagsik[1] made his enormous fortune by exploiting such mechanisms (a version of financial oscillator denoted $sbO_{0.7}$, see below). We would like to argue that this is not justified because the real profits from the use of the oscillator $sbO_{0.7}$ do not result in substantial magnification of the capital. We propose

[1]It was A. Gąsiorowski who first used the term financial oscillator. It is usually referred to as the Bagsik oscillator in Polish literature and we decided to follow this convention although the term Bagsik-Gąsiorowski oscillator is more appropriate.

a different explanation of the Bagsik rapid enrichment based on a different capital source (the appropriate oscillator will be denoted as $sbO_{1.93}$). Due to the polemic character of the article we will refer to numerical data and time spread discussed in Ref. [1].

2 Logarithmic discount rates

To determine discount rates (or interest rates for deposits) in the interval between the moments $k, m \in \mathbb{N}$, $k < m$ banks use a discount factor $U(k,m) \in \mathbb{R}_+$. The lengths of the intervals in question are not necessary the same if measured in physical units of time. This means that the bank lends the amount of 1 at the moment k on return of the amount of $U_c(k,m)$ at the moment m. Analogously, if the amount of 1 is deposited at the moment k then the bank gives back the amount of $U_d(k,m)$ at the moment m. Profitability of bank activities implies the inequality $U_c(k,m) > U_d(k,m)$. The discount factor is a monotone function, $U(m, m+k) > 1$, and fulfills the condition of multiplicativity $U(k,l)U(l,m) = U(k,m)$. Of course, $U(m,m) = 1$. It is convenient to make use of the notion of logarithmic rates $R(k,m) := \ln U(k,m)$ because their properties are more legible and calculations are simpler. The appropriate properties take the form $R(m, m+k) > 0$, $R(m,m) = 0$, and

$$R(k,l) + R(l,m) = R(k,m). \tag{1}$$

3 The slow bond oscillator

Let us suppose that an arbitrageur has at his disposal two banks, A and B. The first one is ready to lend on the basis of the logarithmic rate $R_A(k,m)$. The second one accepts deposits on the basis of the rate $R_B(k,m)$. In addition let $R_A(m, m+k) \ll R_B(m, m+k)$. The arbitrageur aims at borrowing capital from A and depositing the capital in B so that the financial gain will be the highest possible. The authors of Ref. [1] focused their attention on the following algorithm (sbO) which, in their opinion, should explain Bagsik unheard-of financial achievements in Poland during 1990.

moment 0: *The banker A estimates that the assets of X (say his premises) would be worth 1 at the moment N, so he lends him the amount of $e^{-R_A(0,N)}$ (say a mortgage loan). The arbitrageur X is obliged to give back A the amount of 1 at the moment N. The banker B offers for a deposit of 1 at the moment 0 the amount of $e^{R_B(0,N)}$ to be paid at the moment N. This means that the banker B enters into the obligation to pay X*

$$p_0 := e^{R_B(0,N) - R_A(0,N)} \tag{2}$$

at the moment N which is testified by issuing an appropriate bond to X.

moment k: *By accepting the new bond, the banker A finds out that the present revealed assets of X (he already has bonds for previously* *revealed assets) will be worth p_{k-1} at the moment N. Therefore A pays the amount of $e^{-R_A(k,N)}p_{k-1}$ to X. The banker B, following the above rules, accepts the deposit of $e^{-R_A(k,N)}p_{k-1}$ and issues the bond to return the additional amount of*

$$p_k := e^{R_B(k,N)-R_A(k,N)}p_{k-1} \tag{3}$$

to X at the moment N.

The multiple issued by the banker B bond (certificate) allows the arbitrageur X to retrieve the stated amount of money from B. The recurrence formula (3) states the banal fact that to know the figures stated on the k-th bond it is sufficient to multiply the amount from the previous one by the capitalization factor $e^{R_B(k,N)-R_A(k,N)}$. The initial condition (2) leads to $p_{N-1} = e^{\sum_{m=0}^{N-1}\left(R_B(m,N)-R_A(m,N)\right)}$ stated on the last, issued just before end the arbitrage, bond. This is the only one bond that is not forwarded to A. The rest of the issued by B bonds is used for securing the obligations of X with respect to A originated at the moments $k = 1, \ldots, N-1$. If X buys a mortgage pledge from A at the moment N then he has, besides the premises, the funds

$$e^{\sum_{m=0}^{N-1}\left(R_B(m,N)-R_A(m,N)\right)} - 1 \tag{4}$$

at his disposal. It is worth noticing that, the banks A and B may be physically different market institutions. The whole property of X is worth $e^{\sum_{m=0}^{N-1}\left(R_B(m,N)-R_A(m,N)\right)}$ at the moment N so the logarithmic rate of return of the arbitrage is

$$\mathfrak{R}_{BA}(0,N) = \sum_{m=0}^{N-1}\Big(R_B(m,N) - R_A(m,N)\Big). \tag{5}$$

We set it in a different type to denote that $\mathfrak{R}_{BA}$ are not additive. The lack of additivity characterizes all aggressive techniques of arbitrage. The additivity of the rates $R(k,m)$ allows to simplify the formula (5)

$$\begin{aligned} \mathfrak{R}_{BA}(0,N) &= \sum_{m=0}^{N-1}\sum_{k=m}^{N-1}\Big(R_B(k,k+1) - R_A(k,k+1)\Big) && (6) \\ &= \sum_{k=1}^{N} k\Big(R_B(k-1,k) - R_A(k-1,k)\Big). && (7) \end{aligned}$$

If all the intervals are uniformly distributed, that is the differences $R_B(k-1,k) - R_A(k-1,k)$ are equal then $R_B(k-1,k) - R_A(k-1,k) = \frac{1}{N}\Big(R_B(0,N) - R_A(0,N)\Big)$ and the appropriate rate $\overline{\mathfrak{R}}_{BA}(0,N)$ is given by

$$\overline{\mathfrak{R}}_{BA}(0,N) = \frac{N+1}{2}\Big(R_B(0,N) - R_A(0,N)\Big). \tag{8}$$

We will call such oscillators uniform *sbO*s. The assets of X who accomplishes a uniform *sbO* are given by the formula (cf. (4))

$$e^{\frac{N+1}{2}(R_B(0,N)-R_A(0,N))}-1 \tag{9}$$

which is equivalent to the one given in Ref. [1] (Eq. (16)). If $R_B(0,N)$ is the highest available in discussed interval deposit rate then the uniform arbitrage is profitable under the condition that $\overline{\mathfrak{R}}_{BA}(0,N)$ is greater than $R_B(0,N)$ which implies $N > \frac{R_B(0,N)+R_A(0,N)}{R_B(0,N)-R_A(0,N)}$. Let us note that the profit given by (5) may be achieved only if there is a closing warranty (CW), that is a possibility of instantaneous transfer of all bonds and the related capitals at the moment N.

4 Bagsik oscillator

A physicist would probably say that the presented method of arbitrage (*sbO*) resembles less an oscillator than the repeating mechanism of a mysterious heat engine driven by two thermal baths with the temperatures R_B and R_A, respectively. The highest efficiency of such engines (without changing constructions) is reached for $R_B = R_{\min}$ and $R_A = R_{\max}$ that is for the, respectively, lowest and highest rates of return during a given interval. Przystawa and Wolf claim that the mechanism $sbO_{0.8-0.1}$, with the thermal bath in the shape of a deposit in a Polish bank (the currency PLN, the one year rate $R_B = 0.8$) and the reservoir created by a credit (the currency USD and the one year rate $R_B = 0.1$). The exchange ratio of PLN to USD was constant during that time. The two currencies were needed only to show that the accomplishing of the oscillator was impossible without the constancy of exchange ratio. They forgot that there were available more interesting ”financial thermostats” at that moment. The present authors think that the hottest $R_{\max}$ and the coldest $R_{\min}$ rates were offered by the hyper-inflation itself. The average prices of non-edible goods raised by 591.2% according to the official state data [2] which gives the logarithmic rate R_* equal to $\ln 6.912 \simeq 1.93$ (with respect to PLN). The prices of services raised much more: by 780.7% which gives $R_{\max} = \ln 8.807 \simeq 2.18$. The rate $R_A = R_{\min} = 0$ was also available: it was possible not to repay an interest free debt in PLN because the undergoing revolutionary changes Polish law did not offer any mechanism of execution of debts revalued by the inflation rate at that moment. Let us select the prices of non-edible goods as the ”heat source” of the oscillator $R_b = R_*$ (it seems to be difficult to use services for doing this). The so defined oscillator $sbO_{1.93}$ had a closing warranty build-in. The bank A formed sellers and the role of the bank B was performed by a belonging to X firm. X simply put off the due payment for the purchased goods till the moment N. The owned by X firm formed a reservoir of goods and immovables any other activity (e. g. production) was inessential. At the moment N the execution of CW was immediate: one queue formed horrified creditors and a second one formed consumers wanting to get rid of theirs cash. The circle was closed by the lawful deferred payment (one could induce directors of state-owned firms to enter such formally legal but tragic in effects contracts). The generally accessible archive of the Polish internet journal *Donosy* [3] reports that at the beginning of the year 1990 (2 of January) the interests of demand deposits were at the level of 7% a year and the three-tears deposits – 38%. Only at the end of the year (13 of December) the interest rates of the one-year deposits raised to 60%. Therefore if we take that a PLN deposit gave a return of 50% on average in 1990 the number would be overestimated. The logarithmic rate of such deposits was not $R_B(0,N) = 0.8$, as is supposed in the Ref. [1], but $\ln 1.5 \simeq 0.4$. This means that the suggested mechanism led to Bagsik's return described by the oscillator $sbO_{0.3}$ and not by $sbO_{0.7}$. The profits given by the formula (4) for the oscillators $sbO_{0.7}$, $sbO_{0.3}$ and $sbO_{1.93}$ are presented in the Table 1. The first column is also given in the Ref. [1]. Note that for $N = 12$ the return of $sbO_{1.93}$ is about 45 000 times bigger than that of $sbO_{0.3}$!

$R_B(0,N)-R_A(0,N)=$	0.7	0.3	1.93
$N=$ 1	1.0138	0.34986	5.8895
2	1.8577	0.56831	17.084
3	3.0552	0.82212	46.465
4	4.7546	1.1170	123.59
5	7.1662	1.4596	326.01
6	10.588	1.8577	857.34
7	15.445	2.3201	2252.0
8	22.336	2.8574	5912.5
9	32.116	3.4817	15521
10	45.993	4.2070	40740
11	65.686	5.0497	1.0694×10^5
12	93.632	6.0287	2.8069×10^5

Table 1: Profits made from a unit of capital for the arbitrage sbO

5 The slow cash oscillator

An arbitrageur performing sbO wastes a substantial amount of time between the moments $k-1$ and k ($k = 1, \ldots, N-1$) on delivering bonds to the banker A. We will denote the average amount of time needed for this delivery by $\tau_{B\to A}$. The authors of [1] suggest the possibility of realization of an arbitrage sbO in Poland of 1990 if the bank A gives credits in USD and the bank B accepts deposits in PLN. Only the bank B could have operated on the territory of Poland because credits in USD where then unavailable. Therefore, for obvious reasons, the interval $\tau_{B\to A}$ was considerably shorter than $\tau_{A\to B}$ during which the arbitrager X transports, avoiding interference from the more and more suspicious customers, more and more capital from A to B in the shape of goods or cash. (The interval $\tau_{A\to B}$ is equal zero for the discussed in the previous section oscillator $sbO_{1.93}$ because $B = X$.) We will ignore the necessity of showing the source of CW and suppose that it was known to the authors of the Ref. [1]. If $\tau_{A\to B} \gg \tau_{B\to A} \simeq 0$ then the algorithm sbO should be replaced by the following one (scO):

moment 0**:** *The banker A estimates that the assets of X would be worth 1 at the moment N, so he lends him the amount of* $p_0 = e^{-R_A(0,N)}$. The arbitrageur X is obliged to give back A the amount of 1 at the moment N.

moment k**:** *The banker B offers for a deposit of p_{k-1} at the moment k the amount of $e^{R_B(k,N)}p_{k-1}$ in the form of a bond becoming due at the moment N. If $k < N-1$ then the banker A takes this bond as a deposit and pays to X the amount $p_k := e^{R_B(k,N)-R_A(k,N)}p_{k-1}$.*

If we take for granted the existence of CW then by repeating the calculation performed for sbO we easily get the profit made by X from the arbitrage

$$\begin{aligned} & e^{R_B(N-1,N)} \prod_{k=1}^{N-2} e^{R_B(k,N)-R_A(k,N)} e^{-R_A(0,N)} - 1 \\ = \quad & e^{\sum_{k=0}^{N-1}(R_B(k,N)-R_A(k,N))} e^{R_A(N-1,N)-R_B(0,N)} - 1. \end{aligned}$$

The profit is smaller than $\mathfrak{R}_{BA}(0,N)$ because it equals

$$\mathfrak{R}_{BA}(0,N) - (R_B(0,N) - R_A(N-1,N)). \tag{10}$$

In the case of a uniform arbitrage the logarithmic rate of return is $\frac{N+1}{2}(R_B(0,N) - R_A(0,N)) - R_B(0,N) + \frac{R_A(,N)}{N}$. Therefore the hypothetical two-currencies variant of the Bagsik oscillator with CW should result in smaller profits than those of the oscillator $sbO_{0.3}$ (presented in the Table 1). The non-multiplicative capitalization coefficient, $\mathfrak{U}(0,N)$, for $scO_{0.3}$ ($N \leq 12$) is smaller 26-36% than the one corresponding to the arbitrage $sbO_{0.3}$ ($e^{R_A(0,1)-R_B(0,1)} \simeq 0.741$, $e^{R_A(11,12)-R_B(0,12)} \simeq 0.676$). Effectively, the profit is the same as in $sbO_{0.3}$ but shortened by one step. We may consider a whole one-parameter family of arbitrage procedures λO for N full cycles, where $\lambda \in [0,1]$ is the quotient of lengths of the characteristic intervals, $\lambda = \frac{\tau_{A\to B}}{\tau_{B\to A}}$, $\tau_{A\to B} + \tau_{B\to A}$=1. For example, in the case when X obtains from A a letter of credit (a document issued by A authorizing the bearer to draw money from another bank at once) then the intervals $\tau_{A\to B}$ i $\tau_{B\to A}$may be equal (the arbitrage $\frac{1}{2}O$). We have already discussed two representatives of the family λO because $0O = sbO$ and $1O = scO$. Note that the logarithmic rate of return is a decreasing function of λ. This follows from the fact that the greater the λ is, the smaller is the length of the whole time of using the heat reservoir B. So the algorithms sbO and scO give the extreme values of profits possible by carrying out one of the procedures λO.

6 The temperature of an arbitrage

The present authors have proposed to use temperature, that is the Lagrange multiplier T^{-1} as a measure of the financial gain [4]. This parameter allows to compare financial achievements on different market and during different time scales. The thermodynamically conjugated to the temperature entropy allows to measure qualities of a financial expert or adviser. The market analyzed in this paper corresponds to a two level physical system with the energies $-R_{\max}$ end $-R_{\min}$. We assign, according to the maximal entropy principle, to groups of investors achieving equal logarithmic rates of return R a representative canonical ensemble. The temperature T^{-1} of the ensemble is given by the following function of R [4]

$$T_R^{-1} = \ln \frac{R - R_{\min}}{R_{\max} - R}, \tag{11}$$

where $R_{\min}$ and $R_{\max}$ are the lowest and the highest rates in the considered interval, respectively. We may use the formula (11) for all real values of R after fixing the branch of the logarithm. For the rates $\mathfrak{R} \notin [R_{\min}, R_{\max}]$ we get

$$T_{\mathfrak{R}}^{-1} = \ln\Big((-1) \cdot \frac{\mathfrak{R} - R_{\min}}{\mathfrak{R} - R_{\max}}\Big) = i\pi + \ln \frac{\mathfrak{R} - R_{\min}}{\mathfrak{R} - R_{\max}}. \tag{12}$$

Note that contrary to the additive rates case [4] the presently discussed arbitrage should be prized the more the lower the real part of the temperature T^{-1} is. May by we should call financial oscillators only those arbitrages with non-zero imaginary parts of the temperature? If we determine the proposed temperatures for the oscillators $sbO_{0.3}$ (the second column of the Table 1) and $sbO_{1.93}$ (the third column of the Table 1) then we get the results presented in the Table 2.

$R_B(0,N) - R_A(0,N) =$	0.3	1.93
$N =$ 1	-1.8352	2.0438$= T_*^{-1}$
2	-1.3466	$i\pi$+1.3985
3	-0.96825	$i\pi$+0.83187
4	-0.64536	$i\pi$+0.60114
5	-0.35222	$i\pi$+0.47243
6	-0.073428	$i\pi$+0.38968
7	0.20252	$i\pi$+0.33182
8	0.48643	$i\pi$+0.28903
9	0.79113	$i\pi$+0.25606
10	1.13565	$i\pi$+0.22988
11	1.5554	$i\pi$+0.20857
12	2.1375	$i\pi$+0.19089

Table 2: Temperatures T^{-1} of the sbO arbitages

Negative temperatures characterize financial activities unprofitable even on a developed efficient market [4]. It is worth to note that the temperatures T^{-1} lower than $T_*^{-1} = 2.0438$ (see the Table 2) also correspond to disadvantageous achievements because during that period the temperature T_*^{-1} was easily achieved by every citizen of Poland who possessed goods of everyday use (and no local money). Therefore the first column of the Table 1 presents doubtful financial achievements. For $scO_{0.3}$ with CW the profit is positive only if $N = 13$ what call in question the possibility of using this oscillator as a tool in making capital in Poland of the early nineties. And we have neglected the substantial starting and clearing costs of such an arbitrage! It would be interesting to know if and to what extent arbitrages of the type $sbO_{0.3}$ implemented by Polish banks served as a driving force of the hyper-inflation. Such an oscillator might consist in giving credits in Polish złoty and accepting deposits in foreign bills. The inflation was brought under control simultaneously with the exhaustion of foreign currencies savings of the population. It seems that this substantially slackened the inflation.

7 Concluding remarks

There is a well known Polish ex-minister, a professor of physics, who did not notice a deficit of a billion in the department under his control though it was noticed by his sister, a provincial teacher. We remember public guesses concerning the sources of Bagsik's fortune. We hope that our arguments limit the inclination towards drawing hasty conclusions from oversimplified models of financial phenomena.

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